Statistical fluctuations of the parametric derivative of the transmission and reflection coefficients in absorbing chaotic cavities

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Motivated by recent theoretical and experimental works, we study the statistical fluctuations of the parametric derivative of the transmission T and reflection R coefficients in ballistic chaotic cavities in the presence of absorption. Analytical results for the variance of the parametric derivative of T and R, with and without time-reversal symmetry, are obtained for both asymmetric and left-right symmetric cavities. These results are valid for arbitrary number of channels, in completely agreement with the one channel case in the absence of absorption studied in the literature.

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I. INTRODUCTION

In chaotic and/or weak disordered quantum systems, phase coherence gives rise to sample-to-sample fluctuations in the most transport properties with respect to a small perturbation in an external parameter such as an applied magnetic field, the incident energy or the shape of the cavity. Those fluctuations are of universal character and depend only on the symmetry properties, such as the presence or absence of time reversal invariance (TRI), and/or spatial symmetry[2,3,4,5].

In wave scattering experiments, such as microwave and acoustic resonators, absorption is always present. Its influence on the universal transmission fluctuations is rather dramatic[6].

Many works has been devoted to the effect of the absorption in the transmission T and reflection R coefficients of a chaotic cavity[6,7,8,9,10,11,12]. However, the derivative of those coefficients with respect to the external parameter has not been considered in the presence of absorption. Parametric derivatives are very important in the characterization of mesoscopic systems with chaotic classical limit.

Here we study the statistical fluctuations of the parametric derivative of the coefficients T and R, motivated by recent experiments in microwave cavities[10,12]. We consider a chaotic cavity connected to two waveguides with an arbitrary number of channels in the presence of arbitrary absorption, with and without TRI. We address both asymmetric and left-right (LR) symmetric cavities. As external parameter we will take shape deformations. The purpose of the paper is three fold: first, those calculations help us in the understanding of the distribution of the energy derivative of T in the presence of absorption[15]. Second, they can serve as a motivation to extend the analysis to the distribution of the derivative of T with respect to shape deformations. Finally, the experimental data of Ref. 10 can be used to consider energy and shape deformations derivatives of R.

In the absence of absorption the distribution of the parametric conductance derivative was calculated analytically by Brouwer *et al*[13] for a quantum dot with two single-mode point contacts. That distribution has algebraic tails, and in the absence (presence) of TRI it shows a cusp (divergence) at zero; the second moment is finite (infinite). The reflection symmetric case was considered in Ref.[14]. There, the distribution of the parametric derivative diverges logarithmically at zero derivative, the exponent of the algebraic is different to that of the asymmetric case.

The paper is organized as follows. In Sec. II we present the main formal elements used throughout the paper, such as the scattering matrix S and its parametric derivative in the presence of absorption. Sec. II A is dedicated to asymmetric cavities. The Poisson's kernel for S and its application to chaotic scattering in the presence of absorption by means of a phenomenological model is presented; the parametric derivative of S is defined in terms of the Wigner time-delay matrix whose eigenvalues are the proper time-delays, the inverse of them being distributed according to the Laguerre ensemble. The general structure for S and its parametric derivative for cavities with LR symmetry is introduced in Sec. II B. The mean and variance of the parametric velocities for T and R, as well as the correlation between the individuals transmission and reflection coefficients are calculated in Sec. III A in the presence of TRI, whereas in Sec. III B in its absence. Sec. IV is dedicated to LR-symmetric cavities where we calculate the variances of parametric velocities in the presence (absence) of TRI. Finally, we present a summary and conclusions in Sec. V.

II. THE S MATRIX AND ITS PARAMETRIC DERIVATIVE

A. Chaotic scattering by asymmetric cavities in the presence of absorption

The scattering problem of a ballistic cavity connected to two waveguides, each supporting N_1, N_2 transverse propagating modes (see Fig. 1), can be described by the scattering matrix S, which in the stationary case relates the outgoing-wave to the incoming-wave amplitudes[16].

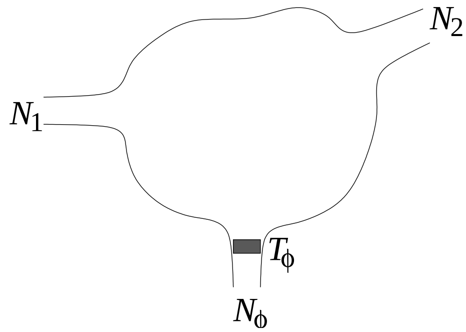

FIG. 1: A ballistic chaotic cavity connected to two leads with N_1, N_2 channels. N_ϕ non transmitting channels are attached to the cavity by tunnel barriers with transmission T_ϕ to model the absorption.

The absorption in the cavity is modeled attaching N_ϕ non transmitting channels to the cavity by means of a tunnel barrier with transmission T_ϕ for each channel[17]. The S matrix is N dimensional ($N = N_1 + N_2 + N_\phi$) with the structure

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{1\phi} \\ s_{21} & s_{22} & s_{2\phi} \\ s_{\phi 1} & s_{\phi 2} & s_{\phi\phi} \end{pmatrix} \equiv \begin{pmatrix} \multicolumn{2}{c}{\widetilde{S}} & s_{1\phi} \\ & & s_{2\phi} \\ s_{\phi 1} & s_{\phi 2} & s_{\phi\phi} \end{pmatrix}, \quad (1)$$

where the set of indices $\{1\}$, $\{2\}$, $\{\phi\}$ label the N_1, N_2, N_ϕ channels. Here, the submatrix $\widetilde{S}$ of dimension N_1+N_2 describes the scattering problem of the absorbing system. The absorption can be quantified by the parameter $\gamma = N_\phi T_\phi$ in the limit $N_\phi \to \infty$, $T_\phi \to 0$ while keeping the product constant[13].

The coefficients of transmission T and reflection R are obtained from the S matrix, actually they depend only on the $\widetilde{S}$ matrix, namely

$$T = \sum_{a\in 1}\sum_{b\in 2} |S_{ab}|^2 \quad \text{and} \quad R = \sum_{a,b\in 1} |S_{ab}|^2 . \quad (2)$$

We restrict ourselves to two of the three basic symmetry classes in the Dyson's scheme[18]. S is always unitary because of flux conservation:

$$SS^\dagger = \mathbb{1}_N , \quad (3)$$

where $\mathbb{1}_N$ stands for the unit matrix of dimension N. (That is not the the case of $\widetilde{S}$ which is a sub-unitary matrix; it represents the scattering matrix of the absorbing system and the flux is not conserved). This case is called "unitary" and it is designated as $\beta = 2$. In addition, in the presence of time reversal invariance S is symmetric,

$$S = S^T . \quad (4)$$

This is the "orthogonal" case, designated as $\beta = 1$.

For systems with a chaotic classical limit, most trasport properties are sample specific and a statistical analysis of the quantum-mechanical problem is appropriate. That study is performed by the construction of ensembles of physical systems, described mathematically by ensembles of S matrices distributed according to a probability law. The starting point is a uniform distribution where S is a member of one of the *circular ensembles*: circular unitary (orthogonal) ensemble, CUE (COE), for $\beta = 2$ ($\beta = 1$)[23].

In the presence of direct processes, the information-theoretic approach of Refs. 19, 20 leads to a S matrix distributed according to the Poisson's kernel[21]

$$P(S) = C \frac{\left[\det\left(\mathbb{1}_N - \langle S\rangle\langle S\rangle^\dagger\right)\right]^{(\beta N+2-\beta)/2}}{\left|\det\left(\mathbb{1}_N - S\langle S\rangle^\dagger\right)\right|^{\beta N+2-\beta}}, \quad (5)$$

where $\langle S\rangle$ is the ensemble averaged S matrix; it describes the prompt response of the system arising from *direct processes*.

A useful construction of the Poisson's ensemble is obtained when the cavity is connected to leads by tunnel barriers[22]. In the case we are concerned with only the fictitious waveguide contains a tunnel barrier, such that the averaged S matrix can be written as

$$\langle S\rangle = \begin{pmatrix} 0_{N_1} & 0 & 0 \\ 0 & 0_{N_2} & 0 \\ 0 & 0 & \sqrt{1-T_\phi}\,\mathbb{1}_{N_\phi} \end{pmatrix} . \quad (6)$$

As before, $\mathbb{1}_n$ stands for the unit matrix of dimensions n and 0_n for the n-dimensional null matrix.

In what follows we restrict ourselves to the case when $T_\phi = 1$, i.e. $P(S)$ is just a constant and the S matrix is uniformly distributed. In such a case, the absorption parameter takes only integer values ($\gamma = N_\phi$), which means moderate and strong absorption. For the present calculations non-integer values of γ are obtained by simple extrapolation.

The parametric derivative of S with respect to a parameter q, that can be the incident energy E or a shape deformation parameter X, is defined as[24]

$$\frac{\partial S}{\partial q} = i\, S^{1/2}\, Q_q\, S^{1/2} \quad (7)$$

where Q_q is an $N\times N$ Hermitian matrix, real symmetric in the presence of time-reversal symmetry. Q_E is proportional to a symmetrized form of the Wigner-Smith time-delay matrix, whose eigenvalues are also the proper delay times. Q_X is defined in analogy to Q_E.

For classically chaotic cavities the joint distribution of S, Q_E and Q_X is[24]

$$P(S, Q_E, Q_X) \propto (\det Q_E)^{-2\beta N - 3(1-\beta/2)} \times \exp\left\{-\beta \,\mathrm{tr}\left[\frac{\pi}{\Delta}Q_E^{-1} + \left(\frac{\pi}{2\Delta}Q_E^{-1}Q_X\right)^2\right]\right\}, \quad (8)$$

where Δ is the mean level spacing and X_c is a typical scale for X. S is independent of Q_E, Q_X and it is uniformly distributed in the space of scattering matrices, i.e. $P(S)$ of Eq. (5) is just a constant. Q_X is Gaussian distributed with a width set by Q_E.

A convenient parametrization for Q_X is given by[24]

$$Q_X = \Psi^{-1\dagger} H \Psi^{-1} \tag{9}$$

where H is a $N \times N$ Hermitian matrix for $\beta = 2$, real symmetric for $\beta = 1$, that have a Gaussian distribution with zero mean and variance

$$\langle H_{ab} H_{cd} \rangle = \begin{cases} 4\,\delta_{ad}\delta_{bc} & \beta = 2 \\ 4\,(\delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}) & \beta = 1 \end{cases} , \tag{10}$$

as can be seen by substitution of Eqs. (9) and (11) into Eq. (8). Ψ is a $N \times N$ matrix, complex in the unitary case, real in the orthogonal one, such that

$$Q_E = \frac{2\pi}{\Delta}\, \Psi^{-1\dagger} \Psi^{-1} \equiv \frac{2\pi}{\Delta}\, Q, \tag{11}$$

where we have defined $Q = \Psi^{-1\dagger}\Psi^{-1}$.

The matrix W of eigenvectors that lead Q to the diagonal form

$$Q = W \hat{\tau} W^{\dagger}, \tag{12}$$

is uniformly distributed in the unitary (orthogonal) group for $\beta = 2$ ($\beta = 1$), independent of S and $\hat{\tau}$. The elements of $\hat{\tau}$, $\{\tau_n\}$ ($n = 1, \ldots, N$), are the proper time delays in dimensionless units; the inverse of them, $x_n = 1/\tau_n$ ($n = 1, \ldots, N$), are distributed according to the Laguerre ensemble[24]

$$P\,(x_1, \ldots, x_N) \propto \prod_{a<b} |x_a - x_b|^{\beta} \prod_{c} x_c^{\beta N/2} e^{-\beta x_c/2} . \tag{13}$$

For the calculations we are interested here, it is convenient to parametrize the S matrix and its parametric derivative as

$$S = UV, \qquad \frac{\partial S}{\partial q} = i\, U Q_q V \quad (q = E,\, X), \tag{14}$$

where U, V are the most general $N \times N$ unitary matrices in the unitary case ($\beta = 2$), while $V = U^T$ in the orthogonal one ($\beta = 1$).

B. Chaotic scattering by symmetric cavities in the presence of absorption

For a system with reflection symmetry, as shown in Fig. 2, the S matrix is block diagonal in a basis of definite parity with respect to reflections, with a circular ensemble in each block[3,4].

In the presence of absorption the S matrix, that describes the scattering of LR ballistic cavity connected to two leads, is of dimension $N = 2N_1 + N_\phi$, where N_1 are the number of channels in each lead (the two leads have the same number of channels and are symmetrically positioned); N_ϕ are the number of absorption channels that

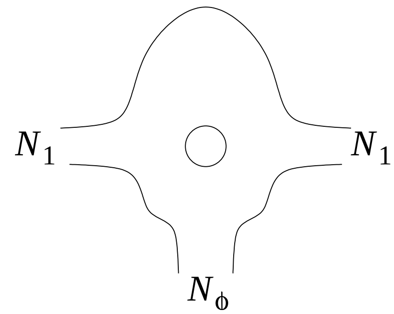

FIG. 2: A ballistic chaotic symmetric cavity connected to two leads supporting N_1 channels. N_ϕ non transmitting channels are attached to the cavity to model the absorption.

we assume symmetrically distributed in the cavity. In that case, the general structure for S is[4]

$$S = \begin{pmatrix} r' & t' \\ t' & r' \end{pmatrix}, \tag{15}$$

where r', t' are $N' \times N'$ matrices, with $N' = N_1 + N_\phi/2$. They represent the reflection and transmission matrices, respectively, associated to the matrix given by (15), and not for the physical cavity. The physical S matrix is a $N_1 \times N_1$ submatrix of that given by Eq. (15).

S-matrices of the form given by Eq. (15), which satisfy (3) are appropriate for systems with reflection symmetry in the absence of TRI. With the additional condition (4) it is appropriate for LR-systems in the presence of TRI. However, when TRI is broken by a uniform magnetic field, the problem of symmetric cavities is mapped[4] to the one of asymmetric cavities with $\beta = 1$ but T replaced by R.

Matrices with the structure (15) can be brought to the block-diagonal form

$$S = R_0^T \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} R_0 \,, \tag{16}$$

where R_0 is the rotation matrix

$$R_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{1}_{N'} & \mathbb{1}_{N'} \\ -\mathbb{1}_{N'} & \mathbb{1}_{N'} \end{pmatrix} . \tag{17}$$

S_1, S_2 are the most general $N' \times N'$ unitary (for $\beta = 2$), and symmetric (for $\beta = 1$) matrices. They have the structure given by Eq. (1) being statistically uncorrelated and uniformly distributed: CUE ($\beta = 2$), COE ($\beta = 1$).

Then, from (16) the transmission and reflection coefficients for LR-symmetric ballistic cavity in the presence of absorption are given by

$$T \;=\; \frac{1}{4} \sum_{a,b=1}^{N_1} |S_{1ab} - S_{2ab}|^2 \quad \text{and} \tag{18}$$

$$R \;=\; \frac{1}{4} \sum_{a,b=1}^{N_1} |S_{1ab} + S_{2ab}|^2 \,, \tag{19}$$

respectively.

The parametric derivative of S is defined through the parametric derivatives of S_1 and S_2 as in Eq. (7). The joint distribution (8) is satisfied for each matrix S_j $j=1$, 2. Finally, they can be parametrized as in Eq. (14).

III. MEAN AND VARIANCE OF THE PARAMETRIC DERIVATIVES OF T AND R FOR ASYMMETRIC CAVITIES

In this section we study the mean and variance of the parametric velocities of the transmission and reflection coefficients, namely, $\partial T/\partial q$ and $\partial R/\partial q$. Here we restrict ourselves to asymmetric cavities, such that T and R are given by Eq. (2). For convenience we define the probability to go from channel b to channel a as

$$\sigma_{ab} = |S_{ab}|^2 \,, \tag{20}$$

such that

$$\frac{\partial T}{\partial q} = \sum_{a\in 1}\sum_{b\in 2}\frac{\partial \sigma_{ab}}{\partial q} \quad \text{and} \tag{21}$$

$$\frac{\partial R}{\partial q} = \sum_{a,b\in 1}\frac{\partial \sigma_{ab}}{\partial q} \,. \tag{22}$$

Substituting the parametrization (14) and using the results of Ref. 26 we average over the matrices U, V for $\beta=2$, or U ($V=U^T$) for $\beta=1$. It is simple to see that the average of $\partial\sigma_{ab}/\partial q$ is zero; hence $\langle \partial T/\partial q\rangle = \langle \partial R/\partial q\rangle = 0$. The fluctuations require a more sophisticated analysis.

We define the correlation coefficient as

$$C_{q_{a'b'}}^{\ ab} = \left\langle \frac{\partial \sigma_{ab}}{\partial q}\frac{\partial \sigma_{a'b'}}{\partial q}\right\rangle, \tag{23}$$

and write the second moments of $\partial T/\partial q$ and $\partial R/\partial q$ as

$$\left\langle \left(\frac{\partial T}{\partial q}\right)^2\right\rangle = \sum_{a,a'\in 1}\sum_{b,b'\in 2} C_{q_{a'b'}}^{\ ab} \tag{24}$$

$$\left\langle \left(\frac{\partial R}{\partial q}\right)^2\right\rangle = \sum_{a,a'\in 1}\sum_{b,b'\in 1} C_{q_{a'b'}}^{\ ab} \,, \tag{25}$$

The $C_{q_{a'b'}}^{\ ab}$ is given by, explicitly written in terms of the S matrix elements,

$$C_{q_{a'b'}}^{\ ab} = 2\mathrm{Re}\left[\left\langle S_{ab}S^*_{a'b'}\frac{\partial S^*_{ab}}{\partial q}\frac{\partial S_{a'b'}}{\partial q}\right\rangle + \left\langle S^*_{ab}S^*_{a'b'}\frac{\partial S_{ab}}{\partial q}\frac{\partial S_{a'b'}}{\partial q}\right\rangle\right]. \tag{26}$$

A. The orthogonal case: $\beta = 1$

1. *Correlation coefficient of $\partial\sigma_{ab}/\partial q$*

First, we calculate the correlation between the different coefficients $\partial\sigma_{ab}/\partial q$. We use the parametrization (14) for $V=U^T$ in Eq. (26) to write

$$C_{q_{a'b'}}^{\ ab} = 2\,\mathrm{Re}\sum_{\alpha,\beta=1}^{N}\sum_{\alpha',\beta'=1}^{N}\left\langle Q_{q_{\alpha\beta}}Q_{q_{\alpha'\beta'}}\right\rangle \times \sum_{c,c'=1}^{N}\left[M\left(\alpha,\beta,c,c\right)-M\left(c,c,\alpha,\beta\right)\right], \tag{27}$$

where for simplicity we define the coefficients

$$M(\alpha,\beta,\gamma,\delta) \equiv M^{a\alpha,b\beta,a'c',b'c'}_{a\gamma,b\delta,a'\alpha',b'\beta'} \equiv \left\langle U_{a\gamma}U_{b\delta}U_{a'\alpha'}U_{b'\beta'}U^*_{a\alpha}U^*_{b\beta}U^*_{a'c'}U^*_{b'c'}\right\rangle. \tag{28}$$

The first (last) two places, α, β, (γ, δ) of the argument on the left-hand side refers to the second and fourth positions in the upper (lower) indices of the coefficients M that appear in Eq. (28). As we can see in App. A, the rest of the indices are not modified in the construction of Eq. (27). Those coefficients M were calculated in Ref. 26 [see Eq. (6.3) of that reference] that we apply to our particular case in App. A. The sum with respect c, c' appearing in the second line of Eq. (27) can be written as

$$\sum_{c,c'=1}^{N}\left[M(\alpha,\beta,c,c)-M(c,c,\alpha,\beta)\right] = -n_1\,\delta^{\beta}_{\alpha}\delta^{\beta'}_{\alpha'} - n_2\,\delta^{\alpha'}_{\alpha}\,\delta^{\beta'}_{\beta} + n_3\,\delta^{\beta'}_{\alpha}\,\delta^{\alpha'}_{\beta}\,, \tag{29}$$

such that Eq. (27) depends on n_1, and n_2, n_3 through the difference $n_3-n_2=Nn_1$ [see Eqs. (A8)-(A10)], which means that n_1 is the only coefficient we need. It is given by (see App. A)

$$\begin{aligned} n_1 &= \frac{1}{N^2(N^2-1)(N+2)(N+3)}\Big\{2(1+\delta^b_a)(1+\delta^{b'}_{a'}) \\ &+ (N+1)\,(N+2)\left(\delta^{a'}_a\,\delta^{b'}_b+\delta^{b'}_a\,\delta^{a'}_b\right)^2 \\ &- (N+1)\Big[\delta^{a'}_b+\delta^{b'}_b+\delta^{a'}_a+\delta^{b'}_a \\ &\quad +2\,\delta^b_a\delta^{b'}_{a'}\left(\delta^{b'}_b\,\delta^a_{a'}+\delta^{a'}_b\,\delta^a_{b'}\right)+2\left(\delta^{b'}_b\,\delta^b_{a'}\delta^{a'}_{b'}\right. \\ &\quad \left.+\ \delta^b_a\delta^{a'}_b\,\delta^a_{a'}+\delta^b_a\delta^{b'}_b\,\delta^a_{b'}+\delta^{b'}_a\,\delta^a_{a'}\,\delta^{a'}_{b'}\right)\Big]\Big\} \end{aligned} \tag{30}$$

Then, Eq. (27) is written as

$$C_{q_{a'b'}}^{\ ab} = 2\,n_1\,\mathrm{Re}\,K_q\,, \tag{31}$$

where

$$K_q = N\sum_{\alpha=1}^{N}\left\langle\left(Q^2_q\right)_{\alpha\alpha}\right\rangle - \sum_{\alpha,\beta=1}^{N}\left\langle Q_{q_{\alpha\alpha}}Q_{q_{\beta\beta}}\right\rangle. \tag{32}$$

K_X is calculated substituting the parametrization (9) into Eq. (32), and performing the average over the matrix H with the help of Eq. (10) for $\beta=1$; the result depends on the Q matrix only, as defined in Eq. (11). K_E is directly written in terms of Q. We write those results in a single equation as

$$K_q = 4\left[(N-2)\,\delta_{qX} + \frac{\pi^2}{\Delta^2}N\delta_{qE}\right]\sum_{\alpha=1}^{N}\left\langle \left(Q^2\right)_{\alpha\alpha}\right\rangle + 4\left[N\delta_{qX} - \frac{\pi^2}{\Delta^2}\delta_{qE}\right]\sum_{\alpha,\beta=1}^{N}\left\langle Q_{\alpha\alpha}Q_{\beta\beta}\right\rangle, \quad (33)$$

where we have introduced the Kronecker's delta $\delta_{qq'}$ to distinguish the result for the parameters X, E. Now, we write Q in its diagonal form as Eq. (12), K_q becomes independent of the unitary matrix W, the average over which is easy to do; the result for K_q depends on only two eigenvalues of Q:

$$K_q = 4N\,(N-1)\left[\left(2\delta_{qX} + \frac{\pi^2}{\Delta^2}\delta_{qE}\right)\left\langle\tau_1^2\right\rangle + \left(N\delta_{qX} - \frac{\pi^2}{\Delta^2}\delta_{qE}\right)\left\langle\tau_1\tau_2\right\rangle\right]. \quad (34)$$

Finally, by direct integration using Eq. (13) for $\beta = 1$, it is simple to see that

$$\left\langle\tau_1^2\right\rangle = \frac{2N!}{(N-2)\,(N+1)!}, \qquad \left\langle\tau_1\tau_2\right\rangle = \frac{(N-1)!}{(N+1)!}. \quad (35)$$

Then, Eq. (34), give us

$$K_X = \frac{4(N-1)N(N+2)}{(N-2)(N+1)}, \qquad K_E = \frac{\pi^2}{\Delta^2}\frac{K_X}{N}. \quad (36)$$

By combination of Eqs. (30), (31), (36), we arrive to the desired results for the correlation coefficients, namely

$$\begin{aligned} C_{X\,a'b'}^{\;ab} &= \frac{8}{(N-2)\,N\,(N+1)^2\,(N+3)} \\ &\times \Big\{2\left(1+\delta_a^b\right)\left(1+\delta_{a'}^{b'}\right) \\ &+ (N+1)\,(N+2)\left(\delta_a^{a'}\delta_b^{b'} + \delta_a^{b'}\delta_b^{a'}\right)^2 \\ &- (N+1)\Big[\delta_b^{a'} + \delta_b^{b'} + \delta_a^{a'} + \delta_a^{b'} \\ &\quad +2\,\delta_a^b\delta_{a'}^{b'}\left(\delta_b^{b'}\delta_{a'}^{a} + \delta_b^{a'}\delta_{b'}^{a}\right) + 2\left(\delta_b^{b'}\delta_{a'}^{b}\delta_{b'}^{a'} \right. \\ &\quad \left. + \,\delta_a^b\delta_b^{a'}\delta_{a'}^{a} + \delta_a^b\delta_b^{b'}\delta_{b'}^{a} + \delta_a^{b'}\delta_{a'}^{a}\delta_{b'}^{a'}\right)\Big]\Big\} \quad (37) \\ C_{E\,a'b'}^{\;ab} &= \frac{\pi^2}{\Delta^2}\frac{C_{X\,a'b'}^{\;ab}}{N}. \quad (38) \end{aligned}$$

The correlation coefficients depend on the absorption parameter N_ϕ through $N = N_1 + N_2 + N_\phi$, that can take integer values only. Extrapolation to an arbitrary absorption is done by replacing N_ϕ by a non-integer number γ, such that $N \to \eta = N_1 + N_2 + \gamma$. For the rest of this subsection we will assume that extrapolation.

From Eqs. (37) and (38) we analyze several cases of interest. From one side, $a' = a \in 1$, $b' = b \in 2$, give the variances of the parametric derivatives of the one channel transmission coefficient $\partial\sigma_{ab}/\partial q$ ($q = X, E$); those are

$$\left\langle\left(\frac{\partial\sigma_{ab}}{\partial X}\right)^2\right\rangle = \frac{8(\eta^2+\eta+2)}{(\eta-2)\eta(\eta+1)^2(\eta+3)} \quad (39)$$

$$\left\langle\left(\frac{\partial\sigma_{ab}}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta}\left\langle\left(\frac{\partial\sigma_{ab}}{\partial X}\right)^2\right\rangle, \quad (40)$$

For very strong absorption they behave as

$$\left\langle\left(\frac{\partial\sigma_{ab}}{\partial X}\right)^2\right\rangle \sim \gamma^{-3}, \qquad \left\langle\left(\frac{\partial\sigma_{ab}}{\partial E}\right)^2\right\rangle \sim \gamma^{-4}. \quad (41)$$

We call this the strongly correlated case.

Now, we present uncorrelated cases. One of them is obtained by making $a' = a \in 1$, $b' \neq b$ ($b, b' \in 2$). In the limit of strong absorption they behave as

$$\left\langle\frac{\partial\sigma_{ab}}{\partial X}\frac{\partial\sigma_{ab'}}{\partial X}\right\rangle \sim \gamma^{-4}, \qquad \left\langle\frac{\partial\sigma_{ab}}{\partial E}\frac{\partial\sigma_{ab'}}{\partial E}\right\rangle \sim \gamma^{-5} \quad (42)$$

that are small compared with Eqs. (41). When all the indices are different they have the behaviour

$$\left\langle\frac{\partial\sigma_{ab}}{\partial X}\frac{\partial\sigma_{a'b'}}{\partial X}\right\rangle \sim \gamma^{-5}, \qquad \left\langle\frac{\partial\sigma_{ab}}{\partial E}\frac{\partial\sigma_{a'b'}}{\partial E}\right\rangle \sim \gamma^{-6}. \quad (43)$$

We conclude that for strong absorption, up to the order of $\langle(\partial\sigma_{ab}/\partial q)^2\rangle$, the correlations between the elements $\partial\sigma_{ab}/\partial q$, for $a \in 1$, $b \in 2$, are very small. Those quantities enter in the construction of $\partial T/\partial q$ [see Eq. (21)] and can be treated as N_1N_2 uncorrelated variables with the same distribution. This is a relevant simplification when the distribution of the parametric derivative of the total transmission coefficient is desired, assuming the one for each $\partial\sigma_{ab}/\partial q$ is known. That is the case of Ref. 15 where the numerical evidence shows an exponential decay for $P(\partial\sigma_{ab}/\partial q)$, $P(\partial T/\partial q)$ being calculated in a very straightforward manner. Eqs. (39), (40) can be used to obtain the decay constant as a function of γ.

2. *Fluctuations of the paremetric derivative of the transmission and reflection coefficients*

The sums of Eqs. (37), (38) with respect to $a, a' \in 1$, $b, b' \in 2$ lead to the variances of the parametric derivatives of T [see Eq. (24)],

$$\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle = 8\frac{N_1N_2\left[(\eta+1)(\gamma+2)+2N_1N_2\right]}{(\eta-2)\eta(\eta+1)^2(\eta+3)} \quad (44)$$

$$\left\langle\left(\frac{\partial T}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta}\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle. \quad (45)$$

Consider the particular case $N_1 = N_2 = 1$ [see Eqs. (39), (40)]. When $\gamma = 0$, the variance of $\partial T/\partial q$ diverges.

This is in agreement with Ref. 13 where the complete distribution of of the parametric derivative of T, in the absence of absorption, was obtained. The distribution has long tails with second moment divergent. We see that absorption suppress that divergence.

For the reflection coefficient we obtain the variances of its parametric derivatives by the sum of Eqs. (37), (38) with respect to a, a', b, $b' \in 1$ [see Eq. (25)]. The results are

$$\left\langle\left(\frac{\partial R}{\partial X}\right)^2\right\rangle = 16\frac{N_1(N_1+1)(\eta-N_1)(\eta-N_1+1)}{(\eta-2)\eta(\eta+1)^2(\eta+3)} \tag{46}$$

$$\left\langle\left(\frac{\partial R}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta}\left\langle\left(\frac{\partial R}{\partial X}\right)^2\right\rangle . \tag{47}$$

We analyze the case of $N_1 = 1$ and $N_2 = 0$, relevant to the experimental data of Ref. 10. In the absence of absorption $\langle(\partial R/\partial q)^2\rangle = 0$ as expected ($R = 1$). For $\gamma = 1$, $\langle(\partial R/\partial q)^2\rangle$ is infinite; making the appropriate correspondence, the distribution for $\partial R/\partial q$ is the same as that obtained in Ref. 13. The divergence in the second moment of $\langle(\partial R/\partial q)^2\rangle$ is suppresed for $\gamma > 1$. It is clear that for $\gamma < 1$ our results do not apply since $\langle(\partial R/\partial q)^2\rangle$ becomes negative.

B. The unitary case: $\beta = 2$

1. Correlation coefficient of $\partial\sigma_{ab}/\partial q$

The unitary case is simpler than the orthogonal one. Following the same procedure, we substitute the parametrization (14) in Eq. (26) to write

$$\begin{aligned} C_{q\,a'b'}^{\;ab} = {} & 2\,\mathrm{Re}\sum_{\alpha,\beta=1}^{N}\sum_{\alpha',\beta'=1}^{N}\Bigg[\left\langle Q_{q_{\beta\alpha}}Q_{q_{\alpha'\beta'}}\right\rangle \\ & \times \sum_{c,c'=1}^{N} M_{ac,a'\alpha'}^{a\alpha,a'c'} M_{cb,\beta'b'}^{\beta b,c'b'} \\ & - \left\langle Q_{q_{\alpha\beta}}Q_{q_{\alpha'\beta'}}\right\rangle \sum_{c,c'=1}^{N} M_{a\alpha,a'\alpha'}^{ac,a'c'} M_{\beta b,\beta'b'}^{cb,c'b'}\Bigg], \end{aligned} \tag{48}$$

where we have defined

$$M_{ab,cd}^{a'b',c'd'} \equiv \langle U'_{ab}U'_{cd}U'^{*}_{a'b'}U'^{*}_{c'd'}\rangle , \tag{49}$$

with U' a unitary matrix (U or V). Those coefficients have been calculated in Ref. 26, and read

$$\begin{aligned} M_{ab,cd}^{a'b',c'd'} = {} & \frac{1}{N^2-1}\Bigg[\left(\delta_a^{a'}\delta_c^{c'}\delta_b^{b'}\delta_d^{d'} + \delta_a^{c'}\delta_c^{a'}\delta_b^{d'}\delta_d^{b'}\right) \\ & - \frac{1}{N}\left(\delta_a^{a'}\delta_c^{c'}\delta_b^{d'}\delta_d^{b'} + \delta_a^{c'}\delta_c^{a'}\delta_b^{b'}\delta_d^{d'}\right)\Bigg]. \end{aligned} \tag{50}$$

After we substitute Eq. (50) into Eq. (48) and sum the dummy inidices

$$C_{q\,a'b'}^{\;ab} = \frac{2\left[1 - N\left(\delta_a^{a'} + \delta_b^{b'}\right) + N^2\delta_a^{a'}\delta_b^{b'}\right]}{N^2(N^2-1)^2}\mathrm{Re}\,K_q\,, \tag{51}$$

where

$$K_q = N\sum_{\alpha=1}^{N}\left\langle(Q_q^2)_{\alpha\alpha}\right\rangle - \sum_{\alpha,\beta=1}^{N}\left\langle Q_{q_{\alpha\alpha}}Q_{q_{\beta\beta}}\right\rangle . \tag{52}$$

To calculate K_X we use of the parametrization (9), perform the average over H that can be done by using Eq. (10). Again, the result depends only on Q. Also, K_E can be written in terms of Q. A single Those results are expressed by a single equation

$$\begin{aligned} K_q = {} & 4\left(-\delta_{qX} + \frac{\pi^2}{\Delta^2}N\delta_{qE}\right)\sum_{\alpha=1}^{N}\left\langle(Q^2)_{\alpha\alpha}\right\rangle \\ & + 4\left(N\delta_{qX} - \frac{\pi^2}{\Delta^2}\delta_{qE}\right)\sum_{\alpha,\beta=1}^{N}\langle Q_{\alpha\alpha}Q_{\beta\beta}\rangle . \end{aligned} \tag{53}$$

Now, we write Q in its diagonal form as in Eq. (12). The result for K_q is independent of the matrix W, the average over wich is easy to do:

$$\begin{aligned} K_q = {} & 4N(N-1)\Bigg[\left(\delta_{qX} + \frac{\pi^2}{\Delta^2}\delta_{qE}\right)\langle\tau_1^2\rangle \\ & + \left(N\delta_{qX} - \frac{\pi^2}{\Delta^2}\delta_{qE}\right)\langle\tau_1\tau_2\rangle\Bigg]. \end{aligned} \tag{54}$$

Again, by direct integration using Eq. (13) we get

$$\langle\tau_1^2\rangle = \frac{2N(N-2)!}{(N+1)!}\,, \qquad \langle\tau_1\tau_2\rangle = \frac{(N-1)!}{(N+1)!}. \tag{55}$$

such that

$$K_X = 4\,N\,, \qquad K_E = 4\frac{\pi^2}{\Delta^2} = \frac{\pi^2}{\Delta^2}\frac{K_X}{N}\,. \tag{56}$$

Eq. (51) lead us to

$$C_{X\,a'b'}^{\;ab} = \frac{8\left[1 - N\left(\delta_a^{a'} + \delta_b^{b'}\right) + N^2\delta_a^{a'}\delta_b^{b'}\right]}{N(N^2-1)^2} \tag{57}$$

$$C_{E\,a'b'}^{\;ab} = \frac{\pi^2}{\Delta^2}\frac{C_{X\,a'b'}^{\;ab}}{N}. \tag{58}$$

In similar manner to the $\beta = 1$ case, from here up to the end of this subsection we make the extrapolation $N_\phi \to \gamma$, such that $N \to \eta = N_1 + N_2 + \gamma$.

We analyze several cases of interest. Firstly, a correlated case is obtained making $a' = a \in 1$, $b' = b \in 2$, which give the variances of $\partial\sigma_{ab}/\partial q$ ($q = X, E$); those are

$$\left\langle\left(\frac{\partial\sigma_{ab}}{\partial X}\right)^2\right\rangle = \frac{8}{\eta(\eta+1)^2} \tag{59}$$

$$\left\langle\left(\frac{\partial\sigma_{ab}}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta}\left\langle\left(\frac{\partial\sigma_{ab}}{\partial X}\right)^2\right\rangle , \tag{60}$$

that for strong absorption they show the behaviour given by Eq. (41).

Uncorrelated cases are obtained when $a' = a \in 1$, $b' \neq b$ $(b, b' \in 2)$, and when all the indices are different: for large γ they go as Eqs. (42), (43), respectively. Those quantities are very small compared to the order of $\langle (\partial \sigma_{ab}/\partial q)^2 \rangle$, meaning that for very large γ the $N_1 N_2$ quatities $\partial \sigma_{ab}/\partial q$ $(a \in 1, b \in 2)$, can be treated as uncorrelated variables with the same distribution $P(\partial \sigma_{ab}/\partial q)$, when the distribution of the parametric derivative of the total transmission coefficient is desired. Numerical evidence[15] also shows a exponential decay of $P(\partial \sigma_{ab}/\partial q)$ for strong absorption; the decay constant depends on γ and on the symmetry class ($\beta = 1$ or 2), and can be obtained from the variance of $\partial \sigma_{ab}/\partial q$.

2. *Fluctuations of the paremetric derivatives of T and R*

We obtain the variances of the parametric derivative of the total transmission coefficient by the sum of Eqs. (57), (58) with respect to the indices a, $a' \in 1$, b, $b' \in 2$, [see Eq. (24)]. The results are

$$\left\langle \left(\frac{\partial T}{\partial X} \right)^2 \right\rangle = \frac{8 N_1 N_2 \left(\eta \gamma + N_1 N_2 \right)}{\eta \left(\eta^2 - 1 \right)^2} \tag{61}$$

$$\left\langle \left(\frac{\partial T}{\partial E} \right)^2 \right\rangle = \frac{\pi^2}{\Delta^2} \frac{1}{\eta} \left\langle \left(\frac{\partial T}{\partial X} \right)^2 \right\rangle \tag{62}$$

Again, consider the case $N_1 = N_2 = 1$ [see Eqs. (59), (60)]. In contrast with the $\beta = 1$ case, $\langle (\partial T/\partial q)^2 \rangle$ does not diverges for $\gamma = 0$. This is in agreement with Ref. 13 too.

Similarly, we get the fluctuations for the parametric derivatives of R from Eq. (25) using Eqs. (57), (58); they are

$$\left\langle \left(\frac{\partial R}{\partial X} \right)^2 \right\rangle = \frac{8 N_1^2 (\eta - N_1)^2}{\eta (\eta^2 - 1)^2} \tag{63}$$

$$\left\langle \left(\frac{\partial R}{\partial E} \right)^2 \right\rangle = \frac{\pi^2}{\Delta^2} \frac{1}{\eta} \left\langle \left(\frac{\partial R}{\partial X} \right)^2 \right\rangle \tag{64}$$

Also, in contrast with the $\beta = 1$ case, $\langle (\partial R/\partial q)^2 \rangle$ does not show any divergence.

IV. VARIANCE OF THE PARAMETRIC DERIVATIVE OF T FOR LEFT-RIGHT SYMMETRIC CAVITIES

In this section we will concentrate on the variance of the parametric transmission velocity, $\partial T/\partial X$ for LR-symmetric cavities. The result for $\langle (\partial T/\partial E)^2 \rangle$ is similar to Eq. (62). Also, because of the LR-symmetry, it is sufficient to consider T, the results for R are equivalent.

For LR-symmetric cavities we define the transmission probability to go from channel a to channel b as (note that in this case a and b can in principle be equal)

$$\begin{aligned} \sigma'_{ab} &= \frac{1}{4} \left| S_{1ab} - S_{2ab} \right|^2 \\ &= \frac{1}{4} \left[\sigma_{1ab} + \sigma_{2ab} - 2 \operatorname{Re} f_{ab} \right], \end{aligned} \tag{65}$$

where the prime on the left hand side indicates that it is defined for the one channel transmission coefficient for LR-symmetric cavities, while σ_1, σ_2 are defined by Eq. (20) and correspond to S_1, S_2 matrices; f_{ab} is an interference term given by

$$f_{ab} = S_{1ab} \, S^*_{2\,ab} \,. \tag{66}$$

Now, the parametric derivative of T cand be written as

$$\frac{\partial T}{\partial X} = \sum_{a,b=1}^{N_1} \frac{\partial \sigma'_{ab}}{\partial X} \tag{67}$$

and its fluctuation as

$$\left\langle \left(\frac{\partial T}{\partial X} \right)^2 \right\rangle = \sum_{a,b=1}^{N_1} \sum_{a',b'=1}^{N_1} C'^{\,ab}_{X\,a'b'} \,, \tag{68}$$

where, in a analogy to Eq. (23), we have defined the correlation coefficient for the symmetric case as

$$C'^{\,ab}_{X\,a'b'} = \left\langle \frac{\partial \sigma'_{ab}}{\partial X} \frac{\partial \sigma'_{a'b'}}{\partial X} \right\rangle . \tag{69}$$

From Eqs. (65) and (69) the correlation coefficient can be expressed as

$$C'^{\,ab}_{X\,a'b'} = \frac{1}{8} \left(C_{1X\,a'b'}^{\;\;ab} + \operatorname{Re} F_{X\,a'b'}^{\;ab} \right), \tag{70}$$

where $C_{1X\,a'b'}^{\;\;ab}$ is given by Eq. (37) for $\beta = 1$, (57) for $\beta = 2$, with N replaced by $N' = N_1 + N_\phi/2$, and

$$F_{X\,a'b'}^{\;ab} = \left\langle \frac{\partial f_{ab}}{\partial X} \frac{\partial f^*_{a'b'}}{\partial X} \right\rangle . \tag{71}$$

To arrive at Eq. (70) we used the fact that S_1, S_2 are uncorrelated but equally and uniformly distributed, such that $C_{2X\,a'b'}^{\;\;ab} = C_{1X\,a'b'}^{\;\;ab}$, $\langle (\partial \sigma_{1ab}/\partial X)(\partial \sigma_{2a'b'}/\partial X) \rangle = 0$, $\langle (\partial \sigma_{j\,ab}/\partial X)(\partial f_{a'b'}/\partial X) \rangle = 0$ for $j = 1$, 2, and $\langle (\partial f_{ab}/\partial X)(\partial f_{a'b'}/\partial X) \rangle = 0$.

Then, it is needed to calculate $F^{ab}_{a'b'}$ that explicitely written in terms of S_1, S_2 matrices elements is

$$\begin{aligned} F_{X\,a'b'}^{\;ab} &= 2 \left[\langle S_{1ab} S^*_{1\,a'b'} \rangle \left\langle \frac{\partial S^*_{2\,ab}}{\partial X} \frac{\partial S_{2\,a'b'}}{\partial X} \right\rangle \right. \\ &+ \left. \left\langle S_{1ab} \frac{\partial S^*_{1\,a'b'}}{\partial X} \right\rangle \left\langle S_{2a'b'} \frac{\partial S^*_{2\,ab}}{\partial X} \right\rangle \right] \end{aligned} \tag{72}$$

We proced to calculate $\langle (\partial T/\partial X)^2 \rangle$ in the presence and absence of TRI. Also, we consider the case when TRI is broken by a magnetic field.

A. Presence of time reversal invariance

1. Correlations of $\partial\sigma'_{ab}/\partial X$

It has been shown in Ref. 14 that the only term of Eq. (72) different from zero is

$$\left\langle \frac{\partial S^*_{j\,ab}}{\partial X}\frac{\partial S_{j\,a'b'}}{\partial X}\right\rangle = \frac{4\left(\delta_a^{a'}\delta_b^{b'}+\delta_a^{b'}\delta_b^{a'}\right)}{N'(N'+1)} \times\left[\sum_{\alpha=1}^{N'}\left\langle\left(Q^2\right)_{\alpha\alpha}\right\rangle+\sum_{\alpha,\beta=1}^{N'}\left\langle Q_{\alpha\alpha}Q_{\beta\beta}\right\rangle\right]. \quad (73)$$

We subtitute Eq. (73) into Eq. (72) and digonalize Q as in Eq. (12). Eq. (13) with N replaced by N', gives $\langle\tau_1\rangle = 1/N'$ by direct integration. Then, we get

$$F_{X\,a'b'}^{\;ab} = \frac{8\left(N'^2+N'+2\right)\left(\delta_a^{a'}\delta_b^{b'}+\delta_a^{b'}\delta_b^{a'}\right)^2}{\left(N'-2\right)N'\left(N'+1\right)^3}. \quad (74)$$

Finally, the last equations together with Eqs. (37), (38) with N' intead of N leads to

$$\begin{aligned} C'^{\;ab}_{X\,a'b'} &= \frac{1}{\left(N'-2\right)N'\left(N'+1\right)^2\left(N'+3\right)} \\ &\times \Big\{2\left(1+\delta_a^b\right)\left(1+\delta_{a'}^{b'}\right) \\ &+ \left(N'+1\right)\left(N'+2\right)\left(\delta_a^{a'}\delta_b^{b'}+\delta_a^{b'}\delta_b^{a'}\right)^2 \\ &- \left(N'+1\right)\Big[\delta_b^{a'}+\delta_b^{b'}+\delta_a^{a'}+\delta_a^{b'} \\ &+2\,\delta_a^b\delta_{a'}^{b'}\left(\delta_b^{b'}\delta_{a'}^{a}+\delta_b^{a'}\delta_{b'}^{a}\right)+2\left(\delta_b^{b'}\delta_{a'}^{b}\delta_{b'}^{a'}\right. \\ &+\left.\delta_a^b\delta_b^{a'}\delta_{a'}^{a}+\delta_a^b\delta_b^{b'}\delta_{b'}^{a}+\delta_a^{b'}\delta_{a'}^{a}\delta_{b'}^{a'}\right)\Big]\Big\} \\ &+ \frac{N'^2+N'+2}{\left(N'-2\right)N'\left(N'+1\right)^3}\left(\delta_a^{a'}\delta_b^{b'}+\delta_a^{b'}\delta_b^{a'}\right)^2. \end{aligned} \quad (75)$$

From here up to the rest of this subsection we will assume the extrapolation to arbitrary absorption: $N_\phi \to \gamma$, $N' \to \eta' = N_1 + \gamma/2$.

Some cases are of particular interest. For simplicity consider only the parametric variation with respect to X. The variance of the parametric derivative of the one-channel transmission coefficient σ'_{aa} is

$$\left\langle\left(\frac{\partial\sigma'_{aa}}{\partial X}\right)^2\right\rangle = \frac{4\eta'\left(\eta'-1\right)}{\left(\eta'-2\right)\eta'\left(\eta'+1\right)^2\left(\eta'+3\right)} + \frac{4\left(\eta'^2+\eta'+2\right)}{\left(\eta'-2\right)\eta'\left(\eta'+1\right)^3}. \quad (76)$$

σ'_{ab} represents a channel to channel transmission coefficient too; its parametric derivative variance is

$$\left\langle\left(\frac{\partial\sigma'_{ab}}{\partial X}\right)^2\right\rangle = \frac{\eta'^2+\eta'+2}{(\eta'-2)\eta'(\eta'+1)^2}\left(\frac{1}{\eta'+3}+\frac{1}{\eta'+1}\right). \quad (77)$$

The two variances are in principle different because at level of the matrices S_1, S_2, σ'_{aa} represents a reflection into the channel a, while σ'_{ab} represents a transmission between the channels a, b. In fact, the first term on the right hand side of Eqs. (76), (77) are the same, except by a factor of 8, as Eqs. (46) (with $N_1 = 1$), (39), respectively, replacing $\eta' \to \eta$. The second term comes from interference between S_1 and S_2 [see Eq. (70)]. However, for strong absorption, $\langle(\partial\sigma'_{aa}/\partial X)^2\rangle$, $\langle(\partial\sigma'_{ab}/\partial X)^2\rangle$, go as γ^{-3}. Up to this order of magnitud, the correlation coefficients for $\partial\sigma_{ab}/\partial X$ $(a, b = 1,\ldots,N_1)$ can be neglected: $\langle(\partial\sigma'_{aa}/\partial X)(\partial\sigma'_{a'b'}/\partial X)\rangle$ and $\langle(\partial\sigma'_{ab}/\partial X)(\partial\sigma'_{ab'}/\partial X)\rangle$ go as γ^{-4}; while $\langle(\partial\sigma'_{aa}/\partial X)(\partial\sigma'_{a'a'}/\partial X)\rangle$, $\langle(\partial\sigma'_{aa}/\partial X)(\partial\sigma'_{a'b'}/\partial X)\rangle$, and $\langle(\partial\sigma'_{ab}/\partial X)(\partial\sigma'_{a'b'}/\partial X)\rangle$ go as γ^{-5}.

We conclude that the variables $\partial\sigma'_{ab}/\partial q$, for a, $b = 1,\ldots,N_1$ are uncorrelated for strong absorption. They enter in the in the construction of $\partial T/\partial q$ [see Eq. (67)], the distribution of which is esyly obtained when the one for $\partial\sigma'_{ab}/\partial q$ is known. That is the case of Ref. 15.

2. Variance of $\partial T/\partial X$

Using Eq. (68) we obtain the variance of the parametric derivative of T. The result for $q = X$, E are

$$\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle = \frac{2N_1(N_1+1)\left(\eta'-N_1\right)\left(\eta'-N_1+1\right)}{\left(\eta'-2\right)\eta'\left(\eta'+1\right)^2\left(\eta'+3\right)} + \frac{2N_1(N_1+1)\left(\eta'^2+\eta'+2\right)}{\left(\eta'-2\right)\eta'\left(\eta'+1\right)^3} \quad (78)$$

$$\left\langle\left(\frac{\partial T}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta'}\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle \quad (79)$$

The effect of the LR-symmetry is clearly noted. The first term of the last equations are similar to Eqs. (46), (47) but for the variance of parametric derivative of R for asymmetric cavities. That is because $\partial T/\partial q$ for LR-symmetric cavity is similar to $\partial R/\partial q$ for asymmetric cavity as can be seen by comparison of Eq. (67) with Eqs. (22). The second term in Eqs. (78), (79) comes from the interference term of matrices S_1, S_2 [see Eq. (70)].

For instance, consider a LR-symmetric chaotic cavity connected to two leads each one supporting one channel ($N_1 = 1$). $\langle(\partial T/\partial q)^2\rangle$ diverges for $\gamma = 2$, but it is finite for $\gamma > 2$. For $0 \le \gamma < 2$ it is not defined.

B. Absence of time reversal invariance

1. Correlations of $\partial\sigma'_{ab}/\partial q$

Again, the only term of Eq. (72) different from zero is[14]

$$\left\langle \frac{\partial S^*_{j\,ab}}{\partial X}\frac{\partial S_{j\,a'b'}}{\partial X}\right\rangle = \frac{4\delta_a^{a'}\delta_b^{b'}}{N'^2}\left[\sum_{\alpha,\beta=1}^{N'}\langle Q_{\alpha\alpha}Q_{\beta\beta}\rangle\right] \qquad (80)$$

Now, we subtitute Eq. (80) into Eq. (72); the digonalization of Q, Eq. (12), gives

$$F_X{}^{ab}_{a'b'} = \frac{8\left(N'^2+1\right)\delta_a^{a'}\delta_b^{b'}}{N'^3\left(N'^2-1\right)}, \qquad (81)$$

where we used Eq. (55) and the result $\langle\tau_1\rangle = 1/N'$ which can be obtained by direct integration from Eq. (13). Finally, Eqs. (81), (57), and (58) with N' intead of N, gives

$$\begin{aligned} C'^{\,ab}_{X\,a'b'} &= \frac{\left[1 - N'\left(\delta_a^{a'} + \delta_b^{b'}\right) + N'^2\delta_a^{a'}\delta_b^{b'}\right]}{N'\left(N'^2-1\right)^2} \\ &+ \frac{\left(N'^2+1\right)\delta_a^{a'}\delta_b^{b'}}{N'^3\left(N'^2-1\right)} \end{aligned} \qquad (82)$$

Again, we assume the extrapolation to arbitrary absorption: $N_\phi \to \gamma$, $N' \to \eta' = N_1 + \gamma/2$.

For simplicity consider only the parametric variation with respect to X. The variances of the parametric derivative of the channel to channel transmission coefficient is given by $\langle(\partial\sigma'_{ab}/\partial X)^2\rangle = \langle(\partial\sigma'_{aa}/\partial X)^2\rangle$,

$$\left\langle\left(\frac{\partial\sigma'_{aa}}{\partial X}\right)^2\right\rangle = \frac{1}{\eta'\left(\eta'+1\right)^2} + \frac{\eta'^2+1}{\eta'^3\left(\eta'^2-1\right)}. \qquad (83)$$

The first term on the right hand side of Eqs. (83) is the same, except by a factor 8, as Eq. (59), replacing $\eta' \to \eta$. The second term comes from interference between S_1 and S_2 [see Eq. (70)]. For strong absorption, $\langle(\partial\sigma'_{aa}/\partial X)^2\rangle$ goes as γ^{-3}. Also, as γ increases the quantities $\partial\sigma'_{ab}/\partial X$, for a, $b = 1,\ldots,N_1$, become uncorrelated.

2. *Variance of $\partial T/\partial q$*

Using Eq. (68) we obtain the variance of the parametric derivative of T. The result are

$$\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle = \frac{N_1^2\left(\eta'-N_1\right)^2}{\eta'\left(\eta'^2-1\right)^2} + \frac{N_1^2\left(\eta'^2+1\right)}{\eta'^3\left(\eta'^2-1\right)} \qquad (84)$$

$$\left\langle\left(\frac{\partial T}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta'}\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle \qquad (85)$$

The effect of the LR-symmetry is clearly noted. The first term of the last equations are similar to Eqs. (63), (64) for the variance of the parametric derivative of R for asymmetric cavities. That is because $\partial T/\partial q$ for LR-symmetric cavity is similar to $\partial R/\partial q$ for asymmetric cavity as can be seen by comparison of Eq. (67) with Eqs. (22). The second term in Eqs. (84), (85) comes from the interference term of matrices S_1, S_2 [see Eq. (70)].

Consider a LR-symmetric chaotic cavity connected to two leads each one supporting one channel ($N_1 = 1$). $\langle(\partial T/\partial q)^2\rangle$ diverges for $\gamma = 0$, but is finite for $\gamma > 0$. This is in contrast with the asymmetric case for $\beta = 2$.

C. TRI broken by a magnetic field

When TRI is broken by a magnetic field, the problem of a LR-symmetric cavity is reduced to the problem of asymmetric cavity with $\beta = 1$ symmetry but the roles of T and R interchanged, such that the parametric derivative of T is given Eq. (22). All the elements $\partial\sigma_{ab}/\partial q$, for a, $b = 1,\ldots,N_1$, are uncorrelated in the strong absorption limit.

In this case, the variance of $\partial T/\partial q$, for $q = X$, E are given by

$$\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle = 16\frac{N_1(N_1+1)(\eta-N_1)(\eta-N_1+1)}{(\eta-2)\eta(\eta+1)^2(\eta+3)} \qquad (86)$$

$$\left\langle\left(\frac{\partial T}{\partial E}\right)^2\right\rangle = \frac{\pi^2}{\Delta^2}\frac{1}{\eta}\left\langle\left(\frac{\partial T}{\partial X}\right)^2\right\rangle. \qquad (87)$$

For a cavity connected to two leads each one supporting one open channel, $\langle(\partial T/\partial q)^2\rangle$ diverges for $\gamma = 0$, also in contrast with the $\beta = 2$ case for asymmetric cavities.

V. SUMMARY AND CONCLUSIONS

The purpose of the present paper has been to study the statistical fluctuations of the derivative, with respect to the incident energy E and shape deformations X, of the transmission T and reflection R coefficients in ballistic chaotic cavities with absorption. We consider asymmetric (Sec. III) and left-right (LR)-symmetric (Sec. IV) systems in the presence and absence of time-reversal invariance (TRI). For all the cases fluctuations of the energy derivative are smaller than those with derivative with respect to shape deformations. For instance, for T we find $\langle(\partial T/\partial E)^2\rangle \propto \langle(\partial T/\partial X)^2\rangle/\eta$, where $\eta = N_1 + N_2 + \gamma$; N_1, N_2 are the number of channels in the waveguides on the left, right, respectively; γ is the absorption strenght. For strong absorption, $\langle(\partial T/\partial E)^2\rangle$ goes to zero faster than $\langle(\partial T/\partial X)^2\rangle$.

In the case of an asymmetric cavity connected to two leads each one with one open channel ($N_1 = N_2 = 1$), at zero absorption, we find that $\langle(\partial T/\partial q)^2\rangle$ ($q = E$, X) is finite when no TRI is present, but is infinite in the presence of TRI. Although our calculations are not valid for weak absorption those results agree with Ref. 13 where a long tails distribution for $\partial T/\partial q$ was obtained. The divergence in the second moment is suppressed by absorption and we expect that the long tails dissapear at sufficiently large γ (see below).

The case of the last paragraph corresponds also to a case of an asymmetric cavity with one-lead-one-channel ($N_1 = 1$, $N_2 = 0$) with one channel of absorption, i.e $\gamma = 1$. In this case, $\langle(\partial R/\partial q)^2\rangle$ is infinite (finite) in the presence (absence) of TRI. The divergence disapear for $\gamma > 1$. $\langle(\partial R/\partial q)^2\rangle = 0$ at zero absorption, as should be, and it is not defined for $0 < \gamma < 1$.

For a left-right (LR)-symmetric cavity connected to two waveguides with one open channel each one ($N_1 = N_2 = 1$), $\langle(\partial T/\partial q)^2\rangle$ is not defined for $0 \le \gamma < 2$, diverges at $\gamma = 2$, and remains finite for $\gamma > 2$ in the presence of TRI. In the absence of TRI, the results are different in the presence or absence of an applied magnetic field. However, in both cases $\langle(\partial T/\partial q)^2\rangle$ diverges at $\gamma = 0$, in contrast with the asymmetric case; $\langle(\partial T/\partial q)^2\rangle$ is finite for $\gamma > 0$. Again, a long tails distribution for $\partial T/\partial q$ is expected at zero absorption for both presence and absence of TRI. Also, we expect the long tails are suppressed at sufficiently strong absorption (see below).

The correlation coefficients for the parametric velocities of the transmission probability σ_{ab} [Eq. (20)] from channel a to channel b, for asymmetric cavities, were calculated. For strong absorption, we found that $\partial\sigma_{ab}/\partial q$, for $a \in 1$, $b \in 2$, are uncorrelated variables. They enter in the construction of $\partial T/\partial q$. That is a relevant simplification when the distribution $P(\partial T/\partial q)$ is desired assuming that $P(\partial\sigma_{ab}/\partial q)$ is known. That is the case of Ref. 15 where a numerical simulation shows an exponential decay for $P(\partial\sigma_{ab}/\partial E)$. The decay constant can be obtained directly from the variance of $\partial\sigma_{ab}/\partial E$: $\langle(\partial T_{ab}/\partial E)^2\rangle = 2/\lambda_\beta^2$. A similar behaviour for $\partial\sigma_{ab}/\partial X$ is expected. This is in contrast with the case of zero absorption where a long tail distribution is obtained for the parametric conductance velocity[13] for a quantum dot connected to two single-channel leads. The same is valid for symmetric cavities.

The results obtained in this paper can serve as a motivation to extend the analysis of Ref. 15 to study the distribution of the transmission derivative with respect to shape deformations, as well as to motivate the analysis of the distribution of the parametric derivative of the reflection coefficient.

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APPENDIX A: THE COEFFICIENTS $M(\alpha,\beta,\gamma,\delta)$

Applying the result (6.3) of Ref. 26 to our case, we can write Eq. (28) as

$$M(\alpha,\beta,\gamma,\delta) = Au_1 + Bu_2 + Cu_3 + Du_4 + Eu_5, \tag{A1}$$

where

$$\begin{aligned}
A &= \frac{N^4 - 8N^2 + 6}{N^2(N^2-1)(N^2-4)(N^2-9)} \\
B &= -\frac{N(N^2-4)}{N^2(N^2-1)(N^2-4)(N^2-9)} \\
C &= \frac{2N^2-3}{N^2(N^2-1)(N^2-4)(N^2-9)} \\
D &= \frac{N^2+6}{N^2(N^2-1)(N^2-4)(N^2-9)} \\
E &= -\frac{5N}{N^2(N^2-1)(N^2-4)(N^2-9)}.
\end{aligned} \tag{A2}$$

and

$$\begin{aligned}
u_1 &= a_1(\delta^{\alpha}_{\gamma}\delta^{\beta}_{\delta}\delta^{c'}_{\alpha'}\delta^{c'}_{\beta'}) + a_2(\delta^{\alpha}_{\gamma}\delta^{c'}_{\delta}\delta^{\beta}_{\alpha'}\delta^{c'}_{\beta'}) \\
&+ a_3(\delta^{\alpha}_{\gamma}\delta^{c'}_{\delta}\delta^{c'}_{\alpha'}\delta^{\beta}_{\beta'}) + a_4(\delta^{\beta}_{\gamma}\delta^{\alpha}_{\delta}\delta^{c'}_{\alpha'}\delta^{c'}_{\beta'}) \\
&+ a_5(\delta^{\beta}_{\gamma}\delta^{c'}_{\delta}\delta^{\alpha}_{\alpha'}\delta^{c'}_{\beta'}) + a_6(\delta^{\beta}_{\gamma}\delta^{c'}_{\delta}\delta^{c'}_{\alpha'}\delta^{\alpha}_{\beta'}) \\
&+ a_7(\delta^{c'}_{\gamma}\delta^{\alpha}_{\delta}\delta^{\beta}_{\alpha'}\delta^{c'}_{\beta'}) + a_8(\delta^{c'}_{\gamma}\delta^{\alpha}_{\delta}\delta^{c'}_{\alpha'}\delta^{\beta}_{\beta'}) \\
&+ a_9(\delta^{c'}_{\gamma}\delta^{\beta}_{\delta}\delta^{\alpha}_{\alpha'}\delta^{c'}_{\beta'}) + a_{10}(\delta^{c'}_{\gamma}\delta^{\beta}_{\delta}\delta^{c'}_{\alpha'}\delta^{\alpha}_{\beta'}) \\
&+ a_{11}(\delta^{c'}_{\gamma}\delta^{c'}_{\delta}\delta^{\alpha}_{\alpha'}\delta^{\beta}_{\beta'}) + a_{12}(\delta^{c'}_{\gamma}\delta^{c'}_{\delta}\delta^{\beta}_{\alpha'}\delta^{\alpha}_{\beta'}),
\end{aligned} \tag{A3}$$

with

$$\begin{array}{ll}
a_1 = 1 + \delta^{b'}_{a'} & a_2 = (1 + \delta^{b'}_{b}\delta^{a'}_{b'})\delta^{a'}_{b} \\
a_3 = (1 + \delta^{a'}_{b}\delta^{b'}_{a'})\delta^{b}_{b'} & a_4 = (1 + \delta^{b'}_{a'})\delta^{b}_{a} \\
a_5 = (\delta^{a'}_{b} + \delta^{b'}_{b}\delta^{a'}_{b'})\delta^{b}_{a}\delta^{a'}_{a} & a_6 = (\delta^{b'}_{b} + \delta^{a'}_{b}\delta^{b'}_{a'})\delta^{b}_{a}\delta^{a}_{b'} \\
a_7 = (\delta^{a'}_{a} + \delta^{b'}_{a}\delta^{a'}_{b'})\delta^{a}_{b}\delta^{b}_{a'} & a_8 = (\delta^{b'}_{a} + \delta^{a'}_{a}\delta^{b'}_{a'})\delta^{a}_{b}\delta^{b}_{b'} \\
a_9 = (1 + \delta^{b'}_{a}\delta^{a'}_{b'})\delta^{a'}_{a} & a_{10} = (1 + \delta^{a'}_{a}\delta^{b'}_{a'})\delta^{b'}_{a} \\
a_{11} = (1 + \delta^{b'}_{a}\delta^{a'}_{b})\delta^{a'}_{a}\delta^{b'}_{b} & a_{12} = (1 + \delta^{a'}_{a}\delta^{b'}_{b})\delta^{b'}_{a}\delta^{a'}_{b}
\end{array}. \tag{A4}$$

The coefficients u_j, for $j = 2,\ldots,5$, are obtained from u_1 through appropiate place permutations of the upper indices (α,β,c',c') of the coefficient M of Eq. (28). u_2 is obtained by the sum of the place permutactions (12), (13), (14), (23), (24), (34), while u_3 by the sum of the permutations (123), (132), (124), (142), (134), (143), (234), (243); u_4 by permutations (12)(34), (13)(24), (14)(23), and finally u_5 by the place permutations (1234), (1243), (1324), (1342), (1423), (1432). The results for u_2, u_3, u_4, u_5 are of the same form as Eq. (A3) but with a_k replaced by coefficients that we call b_k, c_k, d_k, e_k, respectively; they depend on sums of a_k's. We will see below that not all them contribute to Eq. (27); then, we show only the coefficients indexed by $k = 11, 12$ that are important to that equation:

$$b_{11} = a_3 + a_5 + a_8 + a_9 + a_{11} + a_{12}$$

$$
\begin{aligned}
b_{12} &= a_2 + a_6 + a_7 + a_{10} + a_{11} + a_{12} \\
c_{11} &= a_2 + a_3 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} \\
c_{12} &= c_{11} \\
d_{11} &= a_1 + a_4 + a_{12} \qquad \text{(A5)} \\
d_{12} &= a_1 + a_4 + a_{11} \\
e_{11} &= a_1 + a_2 + a_4 + a_6 + a_7 + a_{10} \\
e_{12} &= a_1 + a_3 + a_4 + a_5 + a_8 + a_9 \,.
\end{aligned}
$$

For instance, the result fot $M(\alpha,\beta,\gamma,\delta)$ can be written as

$$
\begin{aligned}
M(\alpha,\beta,\gamma,\delta) &= m_1(\delta^{\alpha}_{\gamma}\delta^{\beta}_{\delta}\delta^{c'}_{\alpha'}\delta^{c'}_{\beta'}) + m_2(\delta^{\alpha}_{\gamma}\delta^{c'}_{\delta}\delta^{\beta}_{\alpha'}\delta^{c'}_{\beta'}) \\
&+ m_3(\delta^{\alpha}_{\gamma}\delta^{c'}_{\delta}\delta^{c'}_{\alpha'}\delta^{\beta}_{\beta'}) + m_4(\delta^{\beta}_{\gamma}\delta^{\alpha}_{\delta}\delta^{c'}_{\alpha'}\delta^{c'}_{\beta'}) \\
&+ m_5(\delta^{\beta}_{\gamma}\delta^{c'}_{\delta}\delta^{\alpha}_{\alpha'}\delta^{c'}_{\beta'}) + m_6(\delta^{\beta}_{\gamma}\delta^{c'}_{\delta}\delta^{c'}_{\alpha'}\delta^{\alpha}_{\beta'}) \\
&+ m_7(\delta^{c'}_{\gamma}\delta^{\alpha}_{\delta}\delta^{\beta}_{\alpha'}\delta^{c'}_{\beta'}) + m_8(\delta^{c'}_{\gamma}\delta^{\alpha}_{\delta}\delta^{c'}_{\alpha'}\delta^{\beta}_{\beta'}) \\
&+ m_9(\delta^{c'}_{\gamma}\delta^{\beta}_{\delta}\delta^{\alpha}_{\alpha'}\delta^{c'}_{\beta'}) + m_{10}(\delta^{c'}_{\gamma}\delta^{\beta}_{\delta}\delta^{c'}_{\alpha'}\delta^{\alpha}_{\beta'}) \\
&+ m_{11}(\delta^{c'}_{\gamma}\delta^{c'}_{\delta}\delta^{\alpha}_{\alpha'}\delta^{\beta}_{\beta'}) + m_{12}(\delta^{c'}_{\gamma}\delta^{c'}_{\delta}\delta^{\beta}_{\alpha'}\delta^{\alpha}_{\beta'}),
\end{aligned} \qquad \text{(A6)}
$$

where

$$
m_k = Aa_k + Bb_k + Cc_k + Dd_k + Ee_k, \quad k = 1,\ldots,12. \qquad \text{(A7)}
$$

From Eq. (A6) we costruct the coefficients $M(\alpha,\beta,c,c)$, $M(c,c,\alpha,\beta)$, take the difference of them and sum with respect to c, c'. The result is given by Eq. (29), where

$$
\begin{aligned}
n_1 &= m_{11} + m_{12} \qquad \text{(A8)} \\
n_2 &= m_2 - m_3 - m_9 + m_{10} - Nm_{11} \qquad \text{(A9)} \\
n_3 &= m_2 - m_3 - m_9 + m_{10} + Nm_{12} \,. \qquad \text{(A10)}
\end{aligned}
$$

Eq. (27) with the help of Eq. (29) leads to Eq. (31), the result being dependent on n_1, and n_2, n_3 through the difference $n_3 - n_2 = Nn_1$; then, we need the coefficient n_1 only. From Eqs. (A8), (A7), (A5), we have

$$
\begin{aligned}
n_1 &= (A + 2B + D)(a_{11} + a_{12}) + 2(D + E)(a_1 + a_4) \\
&+ (B + 2C + E)(a_2 + a_3 + a_5 \\
&+ a_6 + a_7 + a_8 + a_9 + a_{10}). \qquad \text{(A11)}
\end{aligned}
$$

With the help of Eqs. (A2), (A4) n_1 is written as Eq. (30).

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